ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 11 Methods - Test Number 1b

Functions and Graphs

Resource Free

Name: SOLUTIONS **Teacher:** _____

Marks: 30

Time Allowed: 30 minutes

Instructions: You **ARE NOT** allowed any notes or calculators.

You will be supplied with a formula sheet.

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1. Determine the equation of the line of symmetry, the turning point and all intercepts of the parabola given by the equation $y = (2 - x)(2x + 8)$.

$y = -2x^2 - 4x + 16$ ✓ ✓

x-intercepts: $(2,0)$ and $(-4,0)$

y-intercept: $(0,16)$ ✓

Line of symmetry: $x = -1$ ✓

T.P. $(-1,18)$ ✓

[5 Marks]

2. Given that $f(x) = x^2 - ax + 5$, find the value of a if the turning point is (3 , -4).

$-4 = 3^2 - 3a + 5$ ✓

$\Rightarrow \quad -4 = 9 - 3a + 5$

$\Rightarrow \quad 3a = 18 \quad$ Thus $a = 6$ ✓

[2 Marks]

3. The lines $y = 3 - x$ and $y = 3x - 5$ intersect at the point B. Find the equation of the line that is perpendicular to $4y + x = 12$ and that passes through point B.

$3 - x = 3x - 5$

$8 = 4x$

$\therefore \quad x = 2, \; y = 1$ ✓

From $4y + x = 12 \Rightarrow y = -\frac{x}{4} + 3 \quad \therefore m = -\frac{1}{4}$ ✓

$\Rightarrow \perp m = 4$ ✓

$y = 4x + c$

$1 = 8 + c$

$\Rightarrow c = -7$ ✓

$\therefore \quad y = 4x - 7$ ✓

[5 Marks]

4. Find the equation of the parabola in the form $y = ax^2 + bx + c$, that passes through the points (1,-12), (0,-12) and (4,12)

$y = ax^2 + bx + c$

Note: $c = -12$ from the point $(0,-12)$ ✓

$(1,-12): \; -12 = a + b - 12$

$(4,12): \; 12 = 16a + 4b - 12$ } ✓

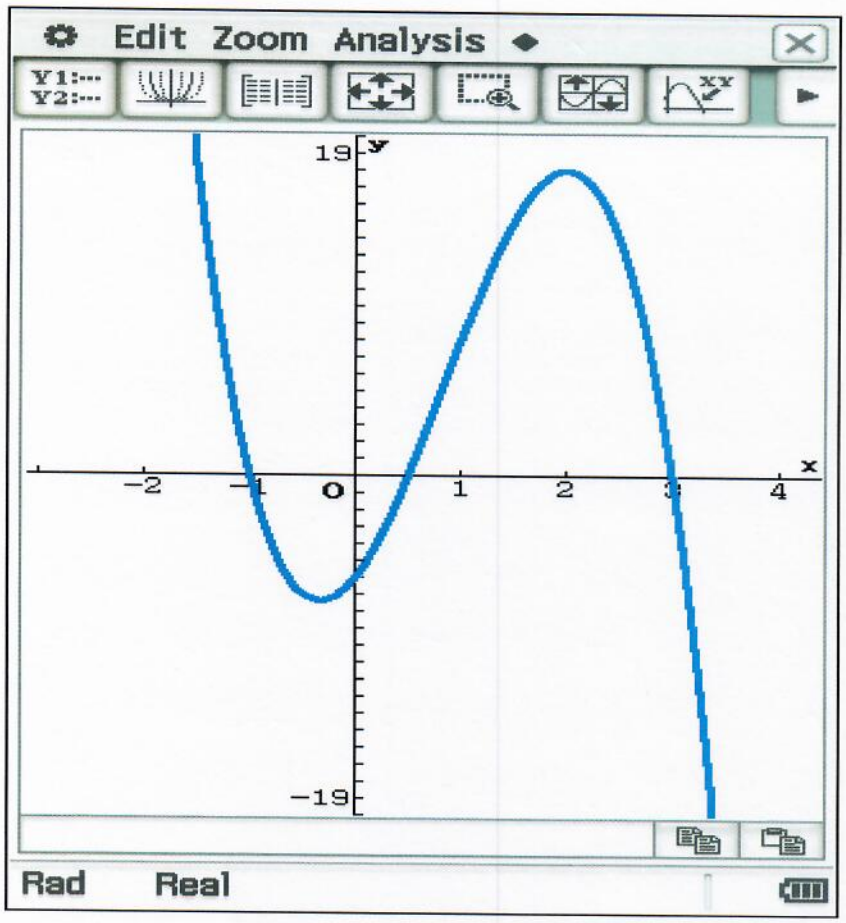
$\Rightarrow \quad 0 = 16a + 16b$

$0 = 16a + 4b - 24$ } ✓ ✓

$\Rightarrow 12b = -24 \quad \therefore b = -2, \; a = 2$ ✓ ✓ Equation $y = 2x^2 - 2x - 12$

[6 Marks]

5. Determine the equation of the cubic function shown below:



Roots: $-1, \frac{1}{2}, 3$

$\Rightarrow a(x+1)(x-\frac{1}{2})(x-3) = y$

From the above $c = \frac{3}{2}a$

$\Rightarrow$ Note y-intercept is -6

$-6 = \frac{3}{2}a$

$-4 = a.$

Thus $y = -4(x+1)(x-\frac{1}{2})(x-3)$

✓ ✓ ✓ ✓

[4 Marks]

6. Choose from the list of functions and relations below:

A $x^2 + y^2 = 100$	**B** $y = x(x+2)^2$	**C** $y = \sqrt{(3x-1)} + 1$	**D** $y = \frac{3}{x+4}$
E $xy = 1$	**F** $y^2 = 13x$	**G** $\frac{2}{x-1} = 3 - y$	**H** $y = x(x-9)$

and **write down only the letter(s)** of all those:

a) which are NOT functions,

A, F

✓✓

-1 MARK FOR INCORRECT GRAPH

b) which represent circles or cubics,

A, B

✓✓

c) whose graphs have domains that exist for all real values,

B, H

✓✓

d) whose graphs have asymptotes.

D, E, G

✓ ✓

ALL 3 FOR 2 MARKS

-1 FOR MISSING OR INCORRECT GRAPHS

[8 Marks]

End of Test

ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 11 Methods - Test Number 1b

Functions and Graphs

Resource Rich

Name: SOLUTIONS **Teacher:** _____

Marks: 20

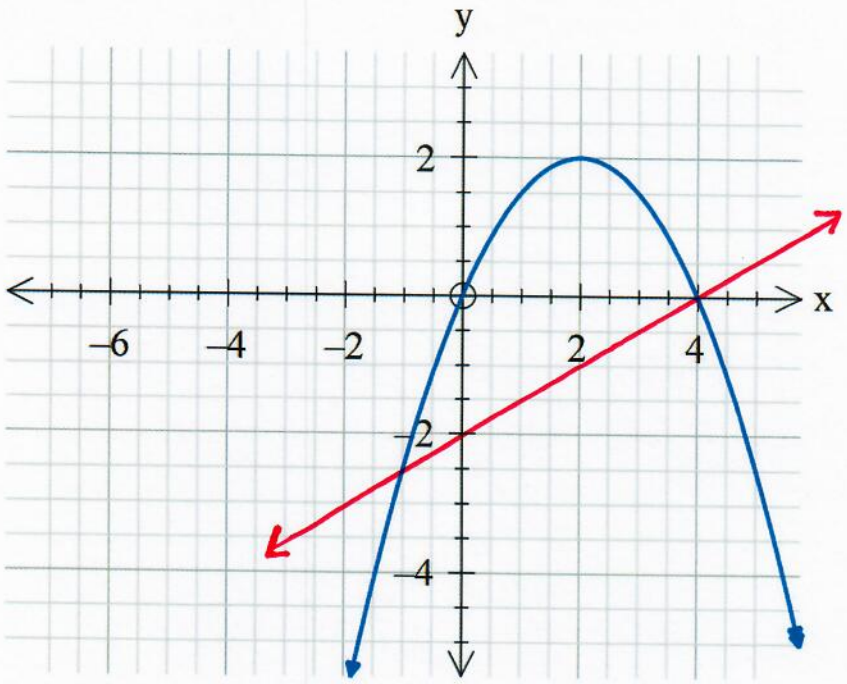
Time Allowed: 15 minutes

Instructions: You **ARE** allowed your calculator(s) but NO NOTES.

You will be supplied with a formula sheet.

1. [2, 2, 2, 2 = 8 marks]

The function $y = f(x)$ is shown below:



(a) State the equation of $f(x)$

$f(x) = -\frac{x}{2}(x-4)$ or $f(x) = -\frac{1}{2}x^2 + 2x$ ✓✓

(b) State the domain and range of $f(x)$

$D = \mathbb{R}$ ✓

$R = y \leq 2$ ✓

Another function is given by $g(x) = 0.5x - 2$

(c) Sketch $y = g(x)$ on the axes above.

(see graph) ✓✓

(d) For what values of x does $f(x) = g(x)$?

4, −1 but accept: ✓✓

(4, 0) and $(-1, -\frac{5}{2})$

2. [4, 4 = 8 marks]

Solve the following equations using the method shown, simplifying your answers where appropriate.

a) $x^2 - 7x = -3$ (by completing the square)

$x^2 - 7x + 3 = 0$

$\Rightarrow \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{12}{4} = 0$ ✓

$\Rightarrow \left(x - \frac{7}{2}\right)^2 - \frac{37}{4} = 0$

$\Rightarrow \left(x - \frac{7}{2} - \frac{\sqrt{37}}{2}\right)\left(x - \frac{7}{2} + \frac{\sqrt{37}}{2}\right) = 0$ ✓

$\therefore x = \frac{7 \pm \sqrt{37}}{2}$ ✓✓ (-1 for 0.459 and 6.541)

b) $-3x^2 - 2 = 5x$ (using the quadratic formula)

$3x^2 + 5x + 2 = 0$

$\Rightarrow a = 3, b = 5, c = 2$ ✓

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-5 \pm \sqrt{25 - 4(3)(2)}}{6}$ ✓

$\Rightarrow \frac{-5 \pm \sqrt{1}}{6}$

$\Rightarrow \frac{-5+1}{6}$ or $\frac{-5-1}{6}$

$\Rightarrow -\frac{2}{3}$ or -1

✓ ✓

3. [4 marks]

Use the discriminant to show that the line $2x - y + 3 = 0$ intersects the circle $x^2 + y^2 = 36$ at two points.

$x^2 + (2x+3)^2 = 36$

$\Rightarrow \quad x^2 + 4x^2 + 12x + 9 - 36 = 0$

$\Rightarrow \quad 5x^2 + 12x - 27 = 0$

$b^2 - 4ac = 144 + 540$

$= +$

Hence: 2 solutions as $\Delta > 0$.

End of Test